Stochastic OPF in presence of Renewable

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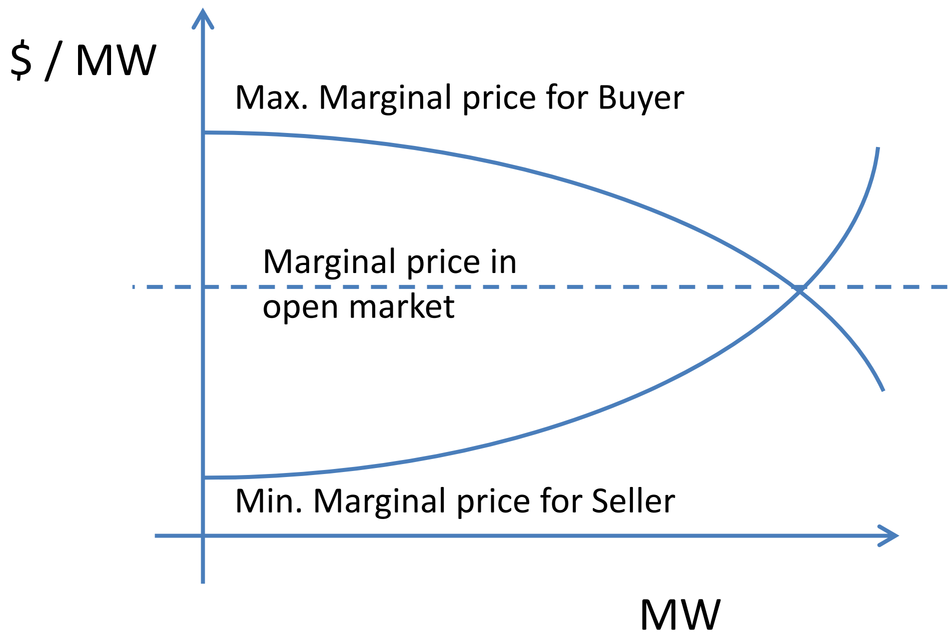
EE292K Intelligent Energy Project

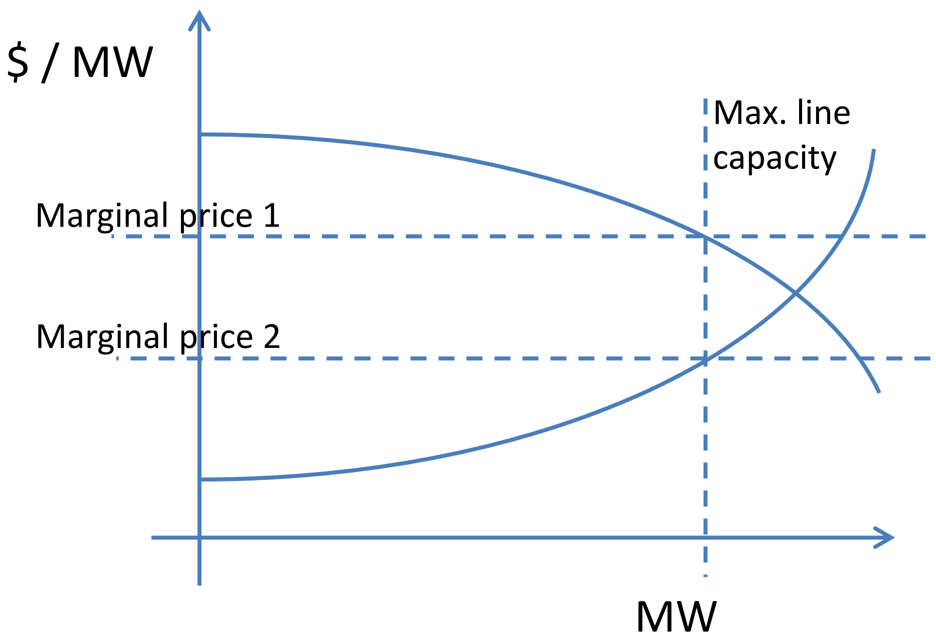
Market

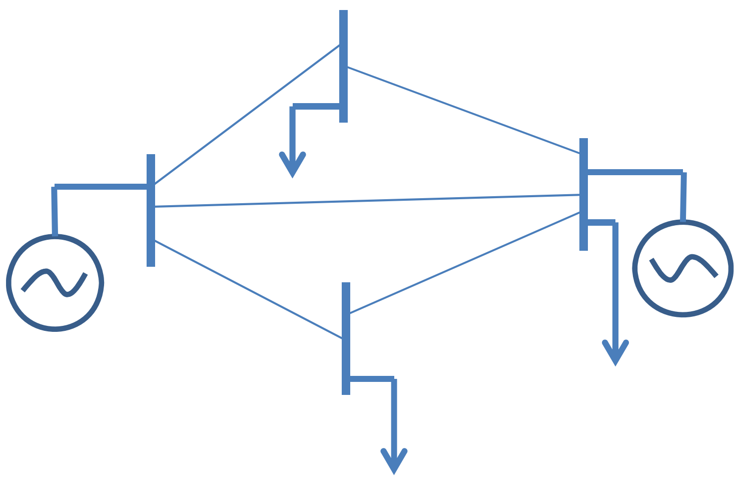
- Buyer's economic utility function
- Seller's cost function
- Open market: Maximize Social Welfare (economic benefit $$)
- Market Price



Electricity Market

- Electrical Energy is bought and sold
- Players – Generator, Load, IGO
- Physical constraints
  - Transmission Line Capacity
  - Power Flow





Optimal Power Flow

$f(P_1, P_2, \ldots, P_n)$ Objective function to minimize

$\underline{V_k} \leq |V_k| \leq \overline{V_k}$ Voltage magnitude limits

$L_{ki} \leq l_{ki}$ Line loss (heat) limits

$\underline{P_k} \leq P_k \leq \overline{P_k}$ Power generation/consumption limit

$P_k = \sum |V_k||V_i|(G_{ki} cos\theta_{ki} + B_{ki} sin\theta_{ki})$ Power Flow

Renewables

- Clean energy generation – reduce carbon footprint
- Low cost to produce (even negative)
- Variable and Unreliable
- Challenges in optimal power flow problem

System Model

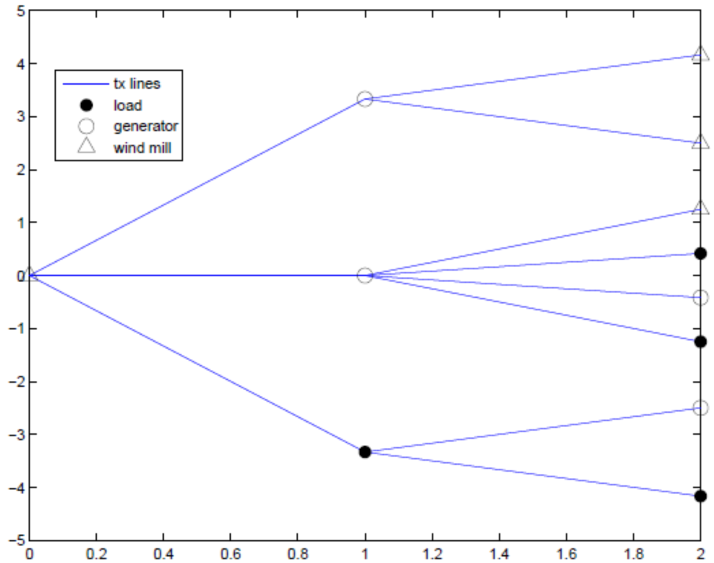


Figure 1: Power system network with $N = 12$ busses (indexed from top to bottom and left to right) and $J = 11$ transmission lines, $B^L = \{4, 8, 10, 12\}$, $B^{NR} = \{2, 3, 9, 11\}$, $B^R = \{1, 5, 6, 7\}$

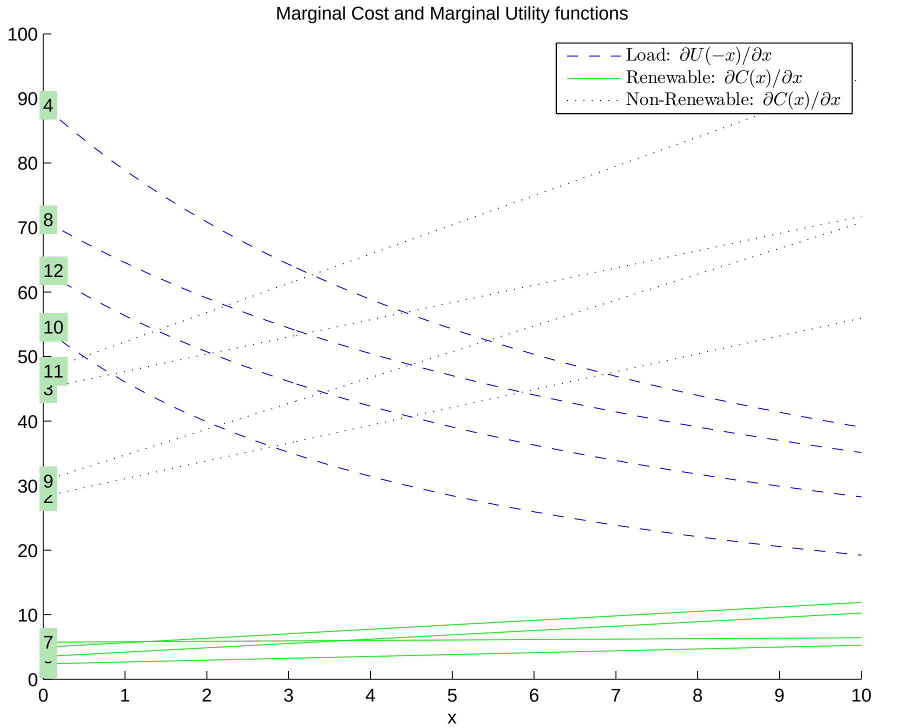


Figure 2: Marginal Cost/Utility functions for generators/loads, labels at start of the curve represent the bus number.

Deterministic Approach

- Maximize social welfare
- Constraints
  - Power flow
  - Line loss
  - Voltage levels
  - Renewable Power
- Not a convex problem
- Linear approximation

$$\underset{\mathbf{V},\mathbf{P}}{\text{minimize}} \quad \sum_{i=1}^{N} C_i(P_i)$$

$$\text{subject to:} \quad P_i = \sum_{k=1}^{N} |V_i||V_k|(G_{ik}cos\theta_{ik} + B_{ik}sin\theta_{ik}),\ i = 1, \ldots, N$$

$$L_i(\mathbf{V}) \leq l^{max},\ i = 1, \ldots, J$$

$$V_{min} \leq |V_i| \leq V_{max},\ i = 1, \ldots, N$$

$$P_i \leq P_i^{max},\ i \in B^R$$

$$P_i \leq 0,\ i \in B^L$$

$$L_{mn}(\mathbf{V}) = |V_m - V_n|^2 g_{mn}$$

How to solve

- Change of Variable
- Convex optimization
- Easy to solve (cvx)
- Restricted to tree topology

$$\mathbf{W} = \mathbf{V}\mathbf{V}^H$$

$$\begin{aligned}
\underset{\mathbf{W},\mathbf{P}}{\text{minimize}} \quad & \sum_{i=1}^{N} C_i(P_i) \\
\text{subject to:} \quad & \mathbf{P} = \Re(diag(\mathbf{W}\mathbf{Y}^H)) \longleftrightarrow \mu \\
& L_i(\mathbf{W}) \leq l^{max}, \ i = 1, \ldots, J \longleftrightarrow \lambda \\
& V_{min}^2 \leq W_{ii} \leq V_{max}^2, \ i = 1, \ldots, N \\
& P_i \leq P_i^{max}, \ i \in B^R \\
& P_i \leq 0, \ i \in B^L \\
& \mathbf{W} \in \mathbb{S}_+^N \\
& [Rank(\mathbf{W}) = 1]
\end{aligned}$$

Solution

- Locational Marginal Prices
- Congestion Cost
- Power Injections

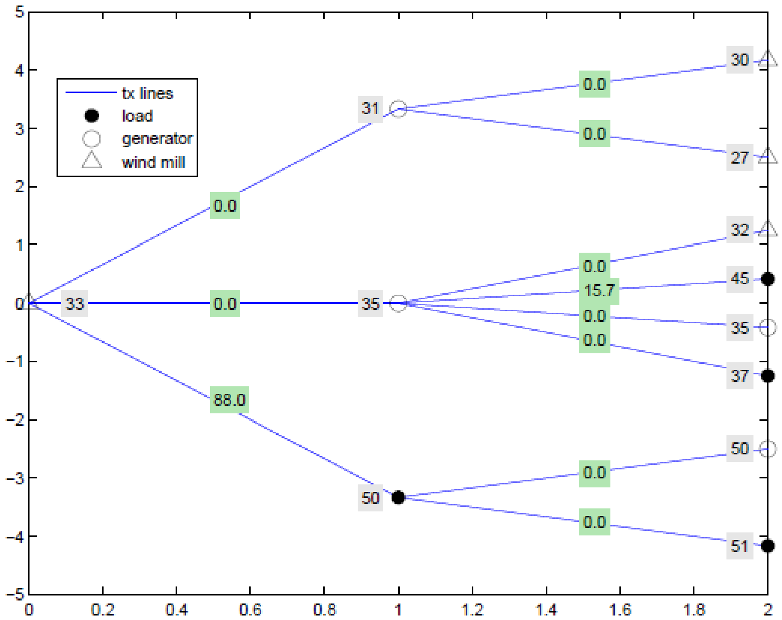


Figure 4: LMP are shown near the bus and congestion cost are shown near center of line

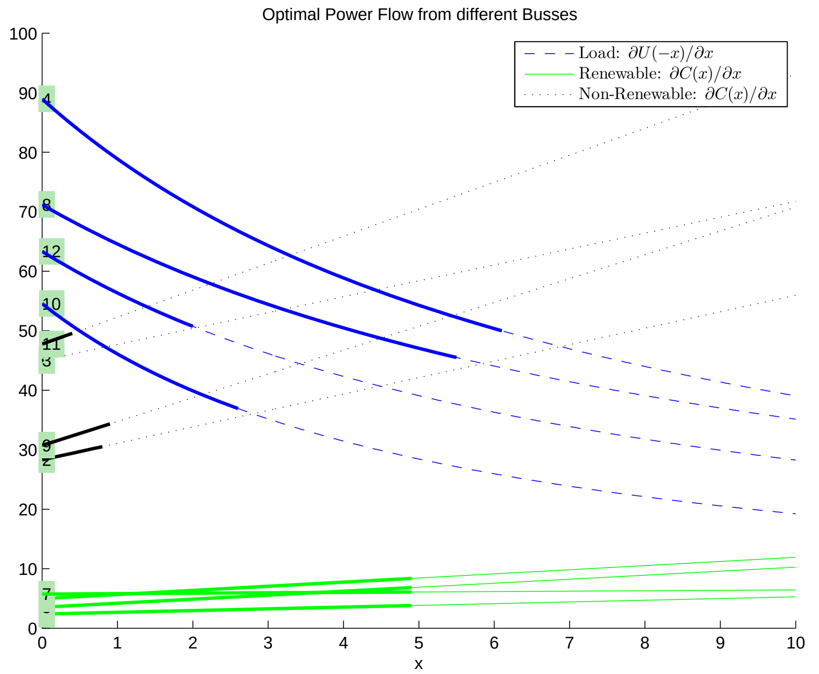


Figure 3: The power injection at various bus has been shown using dark lines on marginal curves.

Stochastic Optimization

- Random variable
- Average social welfare $
- Average line loss
- Knowledge of probability distribution

$$
\begin{aligned}
\underset{\mathbf{W}(\mathbf{X}),\mathbf{P}(\mathbf{X})}{\text{minimize}} \quad & E_{\mathbf{X}}\left(\sum_{i=1}^{N} C_i(P_i(\mathbf{X}))\right) \\
\text{subject to:} \quad & \mathbf{P}(\mathbf{X}) = \Re(diag(\mathbf{W}(\mathbf{X})\mathbf{Y}^H)) \\
& E_{\mathbf{X}}(L_i(\mathbf{W}(\mathbf{X}))) \le l^{max},\ i = 1, \dots, J \longleftrightarrow \lambda \\
& V_{min}^2 \le W_{ii}(\mathbf{X}) \le V_{max}^2,\ i = 1, \dots, N \\
& P_i(\mathbf{X}) \le P_i^{max}(\mathbf{X}),\ i \in B^R \\
& P_i(\mathbf{X}) \le 0,\ i \in B^L \\
& \mathbf{W}(\mathbf{X}) \in \mathbb{S}_+^N
\end{aligned}
$$

Online Learning

- Iterative in time
- Stochastic sub-gradient method
- 'learns' the distribution
- λ converges approx.

$$\begin{aligned}
\underset{\mathbf{W}^t,\mathbf{P}^t}{\text{minimize}} & \sum_{i=1}^{N} C_i(P_i^t) + \sum_{i=1}^{J} \lambda_i^t \left(L_i(\mathbf{W}^t) - l^{max}\right) \\
\text{subject to: } & \mathbf{P}^t = \Re(diag(\mathbf{W}^t \mathbf{Y}^H)) \\
& V_{min}^2 \leq W_{ii}^t \leq V_{max}^2, \; i = 1, \ldots, N \\
& P_i^t \leq P_i^{max,\,t}, \; i \in B^R \\
& P_i^t \leq 0, \; i \in B^L \\
& \mathbf{W}^t \in \mathbb{S}_+^N
\end{aligned}$$

$$\lambda_i^{t+1} = \lambda_i^t + \frac{\alpha}{t}\left(L_i(\mathbf{W}^t) - l^{max}\right)$$

Results

- Line loss variation
- Locational Marginal Prices variation

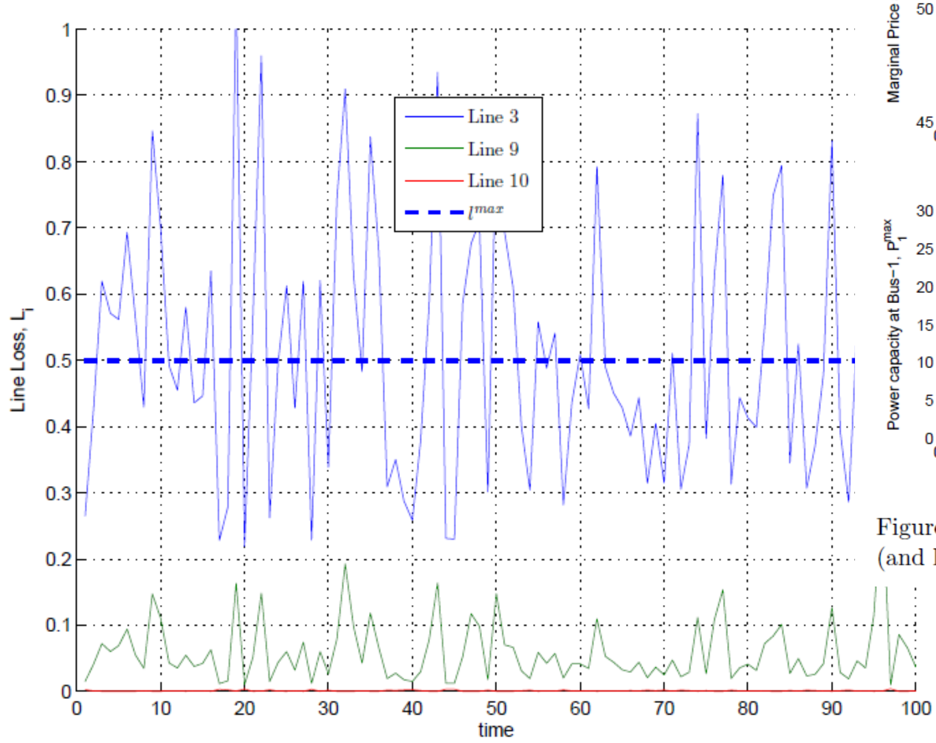


Figure 7: Variation in line loss with time

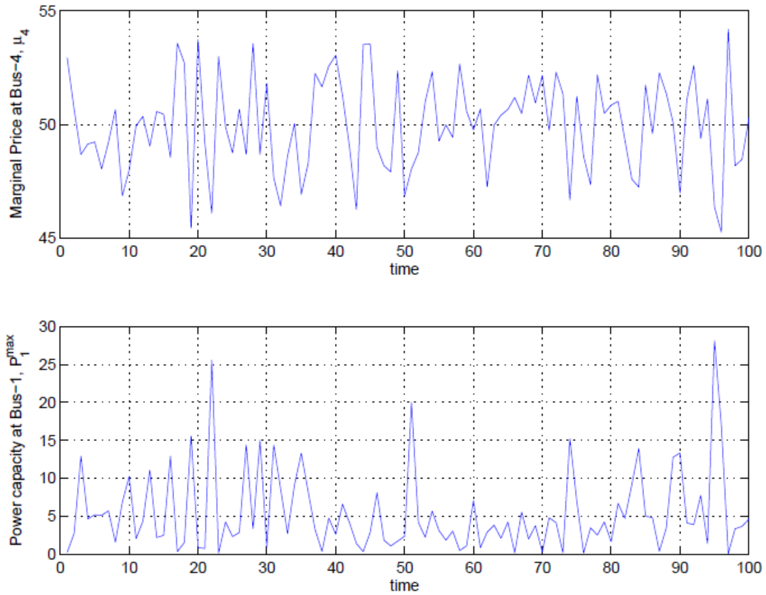


Figure 6: Figure shows the variation of LMP at bus-4 (load bus) with variations of wind (and hence P_1^{max})

Different distribution

- Optimal social welfare decreases with increase in standard deviation
- Congestion cost tends to increase

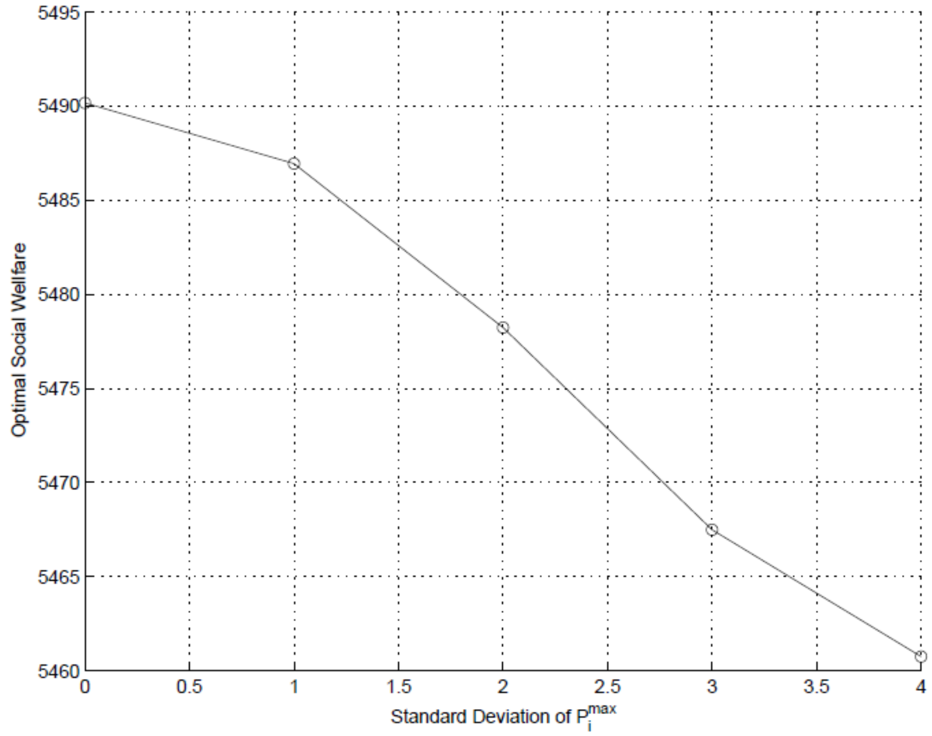
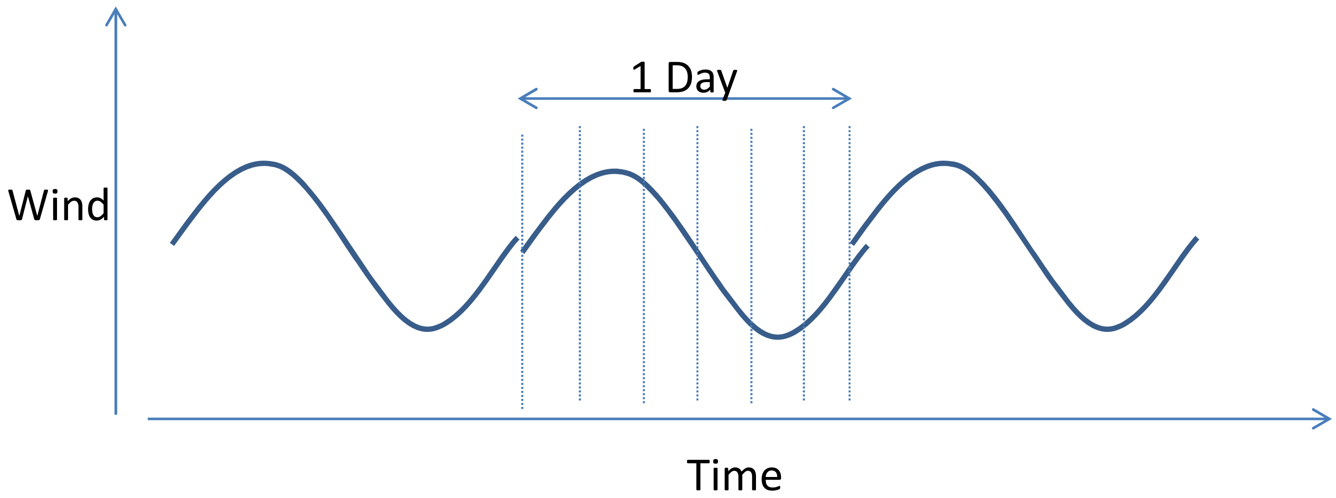


Figure 8: Effect of standard deviation of renewables on social welfare

Diurnal Wind Pattern

- Most variation captured by diurnal pattern
- Optimize over finite number of time slot
- Close to optimal



Rolling Horizon

$$
\begin{aligned}
\underset{\mathbf{W}(t),\mathbf{P}(t)}{\text{minimize}} \quad & \sum_{t=1}^{T}\sum_{i=1}^{N} C_i(P_i(t)) \\
\text{subject to:} \quad & \mathbf{P}(t) = \Re(diag(\mathbf{W}(t)\mathbf{Y}^H)) \; t = 1,\ldots,T \\
& \frac{1}{T}\sum_{t=1}^{T} L_i(\mathbf{W}(t)) \le l^{max} + \delta_{Li}, \; i = 1,\ldots,J \\
& V_{min}^2 \le W_{ii}(t) \le V_{max}^2, \; i = 1,\ldots,N \\
& P_i(t) \le P_i^{max}(t) + \sum_{\tau=1}^{t-1}(P_i^{max}(\tau) - P_i(\tau)) + \delta_{Pi}, \; i \in B^R, \; t = 1,\ldots,T \\
& P_i(t) \le 0, \; i \in B^L, \; t = 1,\ldots,T \\
& \mathbf{W}(t) \in \mathbb{S}_+^N, \; t = 1,\ldots,T
\end{aligned}
$$

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